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$$\begin{aligned}
 &= (ah + bf - ce - dg)^2 + (af - bh - cg + de)^2 \\
 &\quad + (-ae + bg - ch + df)^2 + (ag + be + cf + dh)^2, \\
 &= (ae + bg - cf - dh)^2 + (ag - be - ch + df)^2 \\
 &\quad + (-af + bh - ce + dg)^2 + (ah + bf + cg + de)^2, \\
 &= (af + bg - ch - de)^2 + (ag - bf - ce + dh)^2 \\
 &\quad + (-ah + be - cf + dg)^2 + (ae + bh + cg + df)^2, \\
 &= (ah + bg - ce - df)^2 + (ag - bh - cf + de)^2 \\
 &\quad + (-af + be - cg + dh)^2 + (ae + bf + ch + dg)^2,
 \end{aligned}$$

the sum of four squares in six different ways by combination of letters.

Since the signs of each of these six can form the sum of four squares in eight different ways, the whole number of ways is  $8 \times 6 = 48$  different ways.

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## PROBLEMS.

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46. Proposed by Professor WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics in the Ohio University, Athens, Ohio.

Find  $\theta$  from  $\cos \theta + \cos 3\theta + \cos 5\theta = 0$ .

47. Proposed by LEONARD E. DICKSON, A. M., Fellow in Mathematics, University of Chicago, Chicago, Illinois.

Prove that  $(-1)(-1) = +1$ .




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## GEOMETRY.

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Conducted by B. F. FINKEL, Kidder, Mo. All contributions to this department should be sent to him.

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## SOLUTIONS OF PROBLEMS.

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34. Proposed by T. JOHN COLE, Columbus, Ohio.

A circular field contains 10 acres. A horse is tied to the fence with a rope sufficiently long to graze over one acre. Find length of the rope (1) when the horse is on the inside (2) when he is on the outside of the fence.

Solution by Professor G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

Let  $C$  be the point to which the rope is fastened,  $A$  and  $D$  two points in the circumference to which the horse can graze when on the inside,  $B$  and  $E$  the points in the circumference to which the horse can graze when on the outside. Let  $O$  be the center of the given circle.

Let  $OA = a = \frac{40}{\sqrt{\pi}}$ , the radius of the given circle,

and  $\angle ACO = \theta$ ,  $\angle BCO = \phi$ . Then we have  $AC = 2a\cos\theta$ ,  $BC = 2a\cos\phi$ . The area common to the two circles in the first case  $= a^2(\pi + 2\theta\cos2\theta - \sin2\theta)$ . The area common to the two circles in the second case  $= a^2(\pi + 2\phi\cos2\phi - \sin2\phi)$ .

Therefore the area upon which the animal can graze upon the inside of the circle is

$$a^2(\pi + 2\theta\cos2\theta - \sin2\theta) = \frac{1}{16}\pi a^2 \dots \dots (1).$$

The area upon the outside is

$$4\pi a^2\cos^2\phi - a^2(\pi + 2\phi\cos2\phi - \sin2\phi) = \frac{1}{16}\pi a^2 \dots \dots (1).$$

$$\text{From (1)} \quad 9\pi 2 + 0\theta\cos2\theta - 10\sin2\theta = 0.$$

$$\text{From (2)} \quad 40\pi\cos^2\phi - 20\phi\cos2\phi + 10\sin2\phi = 11\pi.$$

Solving by the method of double position,

$$\theta = 76^\circ 21' 44''.04,$$

$$\phi = 77^\circ 38' 25''.$$

$$AC = 2a\cos\theta = 10.64216 \text{ rods},$$

$$BC = 2a\cos\phi = 9.65892 \text{ rods}.$$

Good solutions to this problem were received from *J. F. W. Scheffer* and *P. S. Berg*.

35. Proposed by LEONARD E. DICKSON, M. A., Fellow in Mathematics, The University of Chicago.

Determine the equation of lowest degree (cubic) upon which depends the inscription of the regular polygon of 37 sides.

**Solution by Professor G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.**

Using the proposers notation as given in his excellent papers in the MONTHLY, we get for  $n=37$ , the following order of subscripts:

1, 2, 4, 8, 16, 5, 10, 17, 3, 6, 12, 13, 11, 15, 7, 14, 9, 18.

Hence the groups are  $(A_1 - A_8 - A_{10} - A_6 + A_{11} - A_{14}) = A$ ,  $(-A_2 - A_{16} + A_{17} - A_{12} + A_{15} + A_9) = B$ ,  $(-A_4 + A_5 + A_3 + A_{13} + A_1 - A_{18}) = C$ .

$$A + B + C = 1. \quad AB = 5(-A_1 - A_8 - A_{10} - A_6 + A_{11} - A_{14})$$

$$-4(-A_2 - A_{16} - A_{17} - A_{12} - A_{15} + A_9)$$

$$-3(-A_4 + A_5 + A_3 + A_{13} + A_1 - A_{18}).$$

$$\therefore AB = -(5A + 4B + 3C) = -5 + B + 2C.$$

$$\text{By symmetry, } AC = -5 + A + 2B, BC = -5 + C + 2A.$$

$$\therefore AB + AC + BC = -15 + 3(A + B + C) = -12.$$

$$ABC = -A(5 - C - 2A) = -A(3 + 2B + C) = -(3A + 2AB + AC).$$

$$\therefore ABC = -(3A - 10 + 2B + 4C - 5 + A + 2B) = -4(A + B + C) + 15.$$

$$\therefore ABC = -11.$$

$\therefore A, B, C$  are the roots of the equation  $x^3 - x^2 + 12x - 11 = 0$ , which is the equation required.

